COMPOUND ANGLES AND PROOF

Using $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$,

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$,

and $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Examples:

1) Expand $\cos(\frac{\pi}{4} + \theta)$

$$\begin{aligned}\cos\left(\frac{\pi}{4} + \theta\right) &= \cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta \\ &= 0.707\cos\theta - 0.707\sin\theta \\ &= \frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta \\ &= \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)\end{aligned}$$

2) Simplify $\cos 2x \cos x + \sin 2x \sin x$

$$\begin{aligned}\cos 2x \cos x + \sin 2x \sin x &= \cos(2x - x) \\ &= \cos x\end{aligned}$$

3) Simplify $\cos 3x \sin 2x - \cos 2x \sin 3x$

$$\begin{aligned}\cos 3x \sin 2x - \cos 2x \sin 3x &= \sin 2x \cos 3x - \cos 2x \sin 3x \\ &= \sin(2x - 3x) \\ &= \sin(-x) \\ &= -\sin x\end{aligned}$$

4) Prove that: $\sin 2x = 2 \sin x \cos x$

LHS = $\sin 2x = \sin(x + x) = \sin x \cos x + \cos x \sin x = 2\sin x \cos x$ = RHS

5) Prove that: $\frac{\cos(A+B)}{\cos A \cos B} = 1 - \tan A \tan B$

LHS = $\frac{\cos(A+B)}{\cos A \cos B} = \frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B} = \frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B} = 1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}$

$= 1 - \tan A \tan B$

$=$ RHS

Delta Ex 34.3 pg 322 Q 4, 5, 6, 14, 16, 18, 28, 29, 30

Delta Ex 34.5 pg 324